Graph theory 5707
Fall Semester 2001
First Midterm
October 17, 2001, 3:35-4:50

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Name:

Comment: Problems 1 through 6 are all of extreme simplicity. In fact, they are all problems of D. West's book that are marked with (–). The last two problems 7 and 8 are somewhat more interesting, but not very hard either. You should plan to spend not more than 5 minutes on the average on each of the problems 1 through 6. Adding five security minutes that makes 35 minutes. The remaining 40 minutes should suffice for the remaining two problems.

#	Points
1	/10
2	/10
3	/10
4	/10
5	/10
6	/10
7	/20
8	/20
Sum	/100
Preliminary grade	

Problem 1 (10 points): Isomorphisms and complements Let G and H be simple graphs. Show that $G \cong H$ if and only if $\bar{G} \cong \bar{H}$.

Problem 2 (10 points): Shrunk matrices Let G be a graph on n vertices. For $v \in V(G)$ and $e \in E(G)$, describe the adjacency matrices of $G - v$ and $G - e$ in terms of the adjacency matrix of G.

Problem 3 (10 points): Euler's edges Prove or disprove: Every Eulerian bipartite graph has an even number of edges.

Problem 4 (10 points): Odd vertices Prove or disprove: If u and v are the only vertices of odd degree in a graph G, then G contains a u, v-path.

Problem 5 (10 points): In cycles Let G be a digraph with no isolated vertices, and in which indegree equals outdegree at each vertex. Prove that G decomposes into (directed) cycles.

Problem 6 (10 points): Different diameters Let G be a simple graph with diameter at least 4. Show that $\bar{G}$ has diameter at most 2.

Problem 7 (20 points): Simply even Prove that the number of simple even graphs with vertex set $\{1, \ldots, n\}$ is $2^{\binom{n-1}{2}}$.
Remember: a graph is called *even*, if every vertex has even degree, and $\binom{n-1}{2}$ is the number of 2-subsets of an $(n-1)$-set.
Hint: Establish a bijection to the set of all simple graphs with vertex set $\{1, \ldots, n-1\}$.

Problem 8 (20 points): Heavy edges Let C be a cycle in a connected weighted graph. Let e be an edge of maximum weight on C.
a) Prove that there is a minimum spanning tree not containing e.
Directions: Consider a minimum weight spanning tree containing e. Show that you can replace e by another edge e' in the cycle C to obtain a minimum weight spanning tree not containing e. For the existence of e' think about what you would get if no $e' \neq e$ in C would work. Then you might want to use the fact that every u, v-walk contains a u, v-path.
b) Use this to prove that you obtain a minimum-weight spanning tree if you iteratively delete a heaviest non-cut edge until the remaining graph is acyclic.
Remark: If you have trouble with part a) just assume it to do part b) then.