Math 5705 Homework #2; due Mon, October 9

Exercise 1: Counting relations Let X be a set on n elements.

a) How many different relations on X are there?

How many of these are ...

b) ... reflexive?
c) ... symmetric?
d) ... antisymmetric?
e) ... reflexive and symmetric?
f) ... reflexive and antisymmetric?

Exercise 2: Cover relations

a) Prove that a partial order on a finite set is uniquely determined by its cover relation.
b) Describe the cover relation for the partial order $\subseteq$ on the collection $\mathcal{P}(X)$ of all subsets of a set X.

Exercise 3: Clock arithmetic Let m be a positive integer and define a relation R on the set X of all nonnegative integers by: aRb if and only if a and b have the same reminder when divided by m. Prove that R is an equivalence relation. How many different equivalence classes does this equivalence relation have?

Exercise 4: Partitions Let Π_n denote the set of all partitions of the set $\{1, \ldots, n\}$. Given two partitions π and σ in Π_n, define $\pi \leq \sigma$, provided each part of π is contained in a part of σ. This relation is usually expressed by saying that π is a *refinement* of σ.

a) Prove that this relation is a partial order on Π_n.
b) Construct the Hasse diagram of $(\Pi_n, \leq)$ for $n = 1, 2, 3$, and 4.

Exercise 4: Mobile integers Prove that in the algorithm that we had in class which generates directly the permutations of $\{1, \ldots, n\}$, the directions of 1 and 2 never change.

Exercise 5: Gray code Build the corners and edges of the 4-cube, and indicate the reflected Gray code on it.

Exercise 6: Lexicographic order Consider the r-combinations of $\{1, \ldots, n\}$ in lexicographic order.

a) What are the first $(n - r + 1)$ r-combinations?
b) What are the last $(r + 1)$ r-combinations?