Math 5705 Homework #4; due November 27, 2000

Exercise 1: Recurrence relation Find and solve a recurrence relation for the number of ternary sequences such that no two zeros are adjacent.
Hint: You might want to determine the number of ternary sequences such that no two zeros are adjacent that end with a zero.

Exercise 2: Inhomogeneous recurrence relations Show that the general solution to any inhomogeneous linear recurrence relation is the general solution to the associated homogeneous relation plus one particular inhomogeneous solution.

Exercise 3: Generating functions Solve the following recurrence relations employing generating functions:

a) $a_n = 3a_{n-1} - 2a_{n-2} + 2$, $a_0 = a_1 = 1$,
b) $a_n = 2a_{n-1} + 2^n$, $a_0 = 1$.

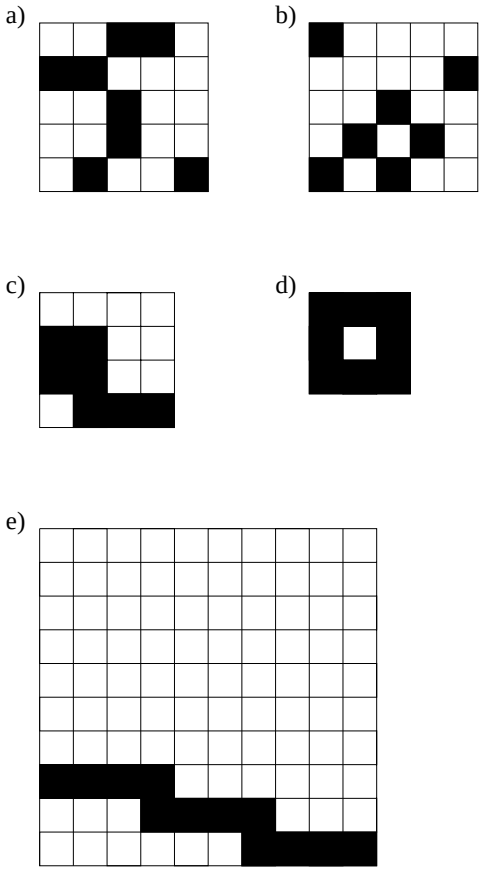
Hint for a): Partial fraction decomposition might be the appropriate thing to do!

Exercise 4: Inclusion/Exclusion How many ways are there to roll 10 distinct dice so that all 6 faces appear?

Exercise 5: Inclusion/Exclusion Given $2n$ letters, two of each of n types, how many arrangements are there with no pair of consecutive letters the same?

Please turn ...

Exercise 6: Rook polynomials Find the rook polynomials for the following boards:



Exercise 7: Ascents An *ascent* in a permutation is a consecutive pair of the form $i, i+1$. The ascents in the following permutation are underlined: 12534.

a) Design a chessboard to represent a permutation of 1,2,3,4,5 so that a check in entry (i, j) means i is followed immediately by j in the permutation. Darken entries so as to exclude all ascents.
b) Find the rook polynomial for the darkened squares in part a).
c) Find the number of permutations of 1,2,3,4,5 with k ascents.