Math 5705 Homework #5; due December 8, 2000

The underlined problems are to be turned in. The others for your practice.

Problem 1: Symmetry of a regular tetrahedron

a) Determine the permutation group of permutations of the vertices of a regular tetrahedron induced by its symmetries.
b) Determine the permutation group of permutations of the edges of a regular tetrahedron induced by its symmetries.
c) Determine the permutation group of permutations of the faces (triangles) of a regular tetrahedron induced by its symmetries.

Problem 2: Symmetry of a rectangle that is not a square Determine the permutation group of permutations of the vertices of a rectangle that is not a square induced by its symmetries.

Problem 3: Applying Burnside Use the Burnside Lemma to determine the number of inequivalent colorings of the corners of a rectangle which is not a square with the colors red and blue.

Problem 4: Two-sided marked dominoes A *two-sided marked domino* consists of two squares joined along an edge where each square on both sides is marked with 0,1,2,3,4,5, or 6 dots.

a) Use the Burnside Lemma to determine the number of different two-sided marked dominoes.
b) How many different two-sided marked dominoes are there if we are allowed to mark the squares with 0,1,2,...,p dots?

Problem 5: Cycle factorization Characterize the cycle factorizations of those permutations π in S_n for which $\pi^{-1} = \pi$.

Problem 6: Cycle factorizations of D_6 Determine the cycle factorization of each element in the dihedral group D_6.

Problem 7: Colorings of a regular 6-gon Determine the number of inequivalent colorings of the vertices of a regular 6-gon with k colors.

Problem 8: More colorings of a regular 6-gon Determine the number of inequivalent colorings of the vertices of a regular 6-gon with 3 colors, where each color is used exactly twice.

Please turn ...

Problem 9: Prime numbers and symmetries Let p be a prime number.

a) Prove that each of the permutations $\rho, \rho^2, \ldots, \rho^{p-1}$ of $\{1, \ldots, p\}$ is a p-cycle, where $\rho = \left(\begin{smallmatrix} 1 & 2 & \cdots & p-1 & p \\ 2 & 3 & \cdots & p & 1 \end{smallmatrix}\right)$.

b) Determine the cycle index of the dihedral group D_{2p}. **Note:** The 2 in D_{2p} is **not** a misprint.

Problem 10: Chessboards again The nine squares of a 3-by-3 chessboard are to be colored red and blue. The chessboard is free to rotate but cannot be flipped over. Determine the generating function for inequivalent colorings and the total number of inequivalent colorings.