Math 5705 Midterm I, October 2, 2000, 1:25 - 2:15

Lecturer: Mark de Longueville

Name:

ID number:

This is a closed book exam. You can obtain scratch paper from me. Explain your answer for each problem step by step. It does not hurt to write some sentences about how you obtained your result. You do not need a calculator. If your result involves numbers that are fairly large and maybe look like

$$P(36; 28, 4, 4) + P(7; 4, 3)$$

or

$$\binom{4}{2} \cdot \binom{85}{7}$$

you don't need to actually compute the number!

1	2	3	4	5	6	7	8	Sum	Grade
/2	/4	/3	/3	/2	/2	/3	/3	/22	

Exercise 1 (2 points): Integers How many integers between 0 and $10,000$ have exactly one digit equal to 5?

Exercise 2 (2+2 points): Typical committee problem A committee of 4 is to be chosen in a math department which has 10 male and 12 female professors (which probably does not exist on this planet).

a) How many ways can the committee be formed if it is to contain at least 2 women?

b) How many ways if, in addition, one particular man and one particular women refuse to serve together on the committee?

Exercise 3 (3 points): Objects Suppose you have nine objects of three types. Type A occurs 3 times, type B occurs 2 times, and type C occurs 4 times. How many 8-permutations of these objects are there?

Exercise 4 (3 points): Classroom situation A classroom has 2 rows of 8 seats each. There are 14 students, 5 of whom always sit in the front row and 4 of whom always sit in the back row. In how many ways can the students be seated?

Exercise 5 (2 points): Integer solutions How many integer solutions of

$$x_1 + x_2 + x_3 + x_4 = 30$$

satisfy $x_1 \geq 2, x_2 \geq 0, x_3 \geq -5$, and $x_4 \geq 8$?

Exercise 6 (2 points): Thirsty students Determine the number of ways to distribute 10 orange drinks, 1 lemon drink, and 1 lime drink to 4 thirsty students so that each student gets at least 1 drink, and the lemon and the lime drinks go to different students.

Exercise 7 (3 points): Three important people Use a combinatorial argument to prove the identity

$$\binom{n}{k} - \binom{n-3}{k} = \binom{n-1}{k-1} + \binom{n-2}{k-1} + \binom{n-3}{k-1}.$$

Hint: Consider a group of n people with three important members A, B, and C.

Exercise 8 (3 points): Walk the block Find a formula for

$$\sum_{\substack{r,s,t\geq 0 \\ r+s+t=n}} \binom{m_1}{r}\binom{m_2}{s}\binom{m_3}{t}$$

where the summation extends over all nonnegative integers r, s, and t with sum $r+s+t=n$. Prove the formula by a block walking argument.