Math 5705 Midterm II, November 10, 2000, 1:25 - 2:15

Lecturer: Mark de Longueville

Name:

1	2	3	4	5	Sum	Grade
/20	/20	/20	/20	/20	/100	

Exercise 1 (20 points): Fruits Determine the generating function for the number b_n of bags of fruit of apples, oranges, bananas, and pears in which there are an even number of apples, at most two oranges, a multiple of three number of bananas, and at most one pear. Then find a formula for b_n from the generating function.

Exercise 2 (20 points): Digits Determine the number d_n of of n-digit numbers with each digit odd where the digits 1 and 3 occur an even number of times.

Exercise 3 (20 points): Sums Let $(a_k)_{k\geq 0}$ be a sequence of numbers with generating function $f(x) = \sum_{k=0}^{\infty} a_k x^k$. Find a generating function for the sequence $(s_r)_{r\geq 0}$ defined by

$$s_r = \sum_{k=0}^{r} k \cdot a_k.$$

Use your result to compute $\sum_{k=0}^{r} \frac{1}{2} k^2$.

Exercise 4 (20 points): Flags Find and solve a recurrence relation for the number f_n of ways to arrange flags on an n-foot flagpole using three types of flags: red flags 2 feet high, yellow flags 1 foot high, and blue flags 1 foot high.

Exercise 5 (20 points): Chessboards Find a recurrence relation for the number c_n of different square subboards of any size that can be drawn on an $n \times n$ chessboard. Solve the recurrence relation by using generating functions.
Hint: Observe that in the following picture each dot determines a unique square touching one or both thick lines.