Linear and combinatorial optimization - 5711
Midterm II
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Name:

#	Points
1	/30
2	/50
3	/20
Sum	/100
Grade	

Problem 1 (30 points): Oil refinery
An oil refinery produces three types of raw gasolines with performance numbers (PN) and daily production levels as given by the following table.

Type	PN	Barrels Produced
Alkylate	107	4180
Catalytic-cracked	93	2512
Straight-run	87	3000

These gasolines can be sold either raw, at \$5 per barrel, or blended into aviation gasolines of two different types Avgas A and Avgas B. Quality standards and prices are given by the following table.

Type	PN	Price per barrel
Avgas A	at least 100	\$7
Avgas B	at least 91	\$6

The performance number (PN) of each mixture is simply the weighted average of the performance numbers of its constituents.
Example: If you mix 150 barrels alkylate, 50 barrels catalytic-cracked, and 200 barrels straight-run gasolines, then you obtain a mixture with performance number

$$\frac{150 \times 107 + 50 \times 93 + 200 \times 87}{150 + 50 + 200} = 95.25.$$

Note that this corresponds to an equality that is linear in the variables specifying the mix:

$$150 \times 107 + 50 \times 93 + 200 \times 87 = 95.25 \times (150 + 50 + 200).$$

The maximal demand for aviation gas is 8000 barrels in total, the refinery can not sell more than that. The refinery aims for the plan that yields the largest possible profit.

(1) Formulate as an LP problem in standard form.
(2) Describe how to compute the total profit.

Instructions:

- Use the variables $x_1, \dots, x_6$ according to the following table.

Variable	Meaning
x_1	barrels of Alkylate used for Avgas A
x_2	barrels of Alkylate used for Avgas B
x_3	barrels of Catalytic-cracked used for Avgas A
x_4	barrels of Catalytic-cracked used for Avgas B
x_5	barrels of Straight-run used for Avgas A
x_6	barrels of Straight-run used for Avgas B

- When defining the objective function remember the meat packing plant problem: what did the objective function mean in this case, how did it relate to the “do-nothing-situation”.
- You will need the following types of inequalities:
  - two for the quality standards of each aviation gas (Let’s call these constraints PN-A and PN-B),
  - three given by the production constraints (ALK, CAT, and STR),
  - one given by the maximal demand for aviation gas (MAXD),
  - non-negativity constraints.

Please use the table on the next page for your final answer. It will also help you for the next problems.

$$
\begin{array}{lcccccccccccccl}
\max & \underline{\quad} & x_1+ & \underline{\quad} & x_2+ & \underline{\quad} & x_3+ & \underline{\quad} & x_4+ & \underline{\quad} & x_5+ & \underline{\quad} & x_6 & & \qquad\text{Type} \\
(1) & \underline{\quad} & x_1+ & \underline{\quad} & x_2+ & \underline{\quad} & x_3+ & \underline{\quad} & x_4+ & \underline{\quad} & x_5+ & \underline{\quad} & x_6 & \leq & \underline{\qquad} \\
(2) & \underline{\quad} & x_1+ & \underline{\quad} & x_2+ & \underline{\quad} & x_3+ & \underline{\quad} & x_4+ & \underline{\quad} & x_5+ & \underline{\quad} & x_6 & \leq & \underline{\qquad} \\
(3) & \underline{\quad} & x_1+ & \underline{\quad} & x_2+ & \underline{\quad} & x_3+ & \underline{\quad} & x_4+ & \underline{\quad} & x_5+ & \underline{\quad} & x_6 & \leq & \underline{\qquad} \\
(4) & \underline{\quad} & x_1+ & \underline{\quad} & x_2+ & \underline{\quad} & x_3+ & \underline{\quad} & x_4+ & \underline{\quad} & x_5+ & \underline{\quad} & x_6 & \leq & \underline{\qquad} \\
(5) & \underline{\quad} & x_1+ & \underline{\quad} & x_2+ & \underline{\quad} & x_3+ & \underline{\quad} & x_4+ & \underline{\quad} & x_5+ & \underline{\quad} & x_6 & \leq & \underline{\qquad} \\
(6) & \underline{\quad} & x_1+ & \underline{\quad} & x_2+ & \underline{\quad} & x_3+ & \underline{\quad} & x_4+ & \underline{\quad} & x_5+ & \underline{\quad} & x_6 & \leq & \underline{\qquad} \\
 & & x_1, & & x_2, & & x_3, & & x_4, & & x_5, & & x_6 & \geq & \underline{\qquad}
\end{array}
$$

The total profit is computed by adding $\underline{\hspace{6em}}$ to ....

Problem 2 (50 points): Production plan
Assume that the management of the refinery established the following production plan.

- Blend 4,061.6 barrels of alkylate, 2,512.0 barrels of catalytic-cracked, and 834.4 barrels of straight-run into aviation gas A.
- Blend 118.4 barrels of alkylate and 473.6 barrels of straight-run into aviation gas B.
- Sell 1,692.0 barrels of raw straight-run gasoline.

(1) What is the total profit with this strategy.
(2) Find out whether this strategy is optimal or not.

Remark: If you feel uncertain about your solution of Problem 1, you should explain very detailed what you are doing in this problem.

Values of the variables for the strategy above:

$x_1^* =$ $\qquad\qquad$ $x_2^* =$
$x_3^* =$ $\qquad\qquad$ $x_4^* =$
$x_5^* =$ $\qquad\qquad$ $x_6^* =$

Help (since you only have 50 minutes...): For this particular solution only one slack variable has a non-zero value caused by selling 1,692.0 barrels of straight-run gasoline raw. Furthermore, the shadow price for catalytic-cracked gasoline is equal to $\frac{6}{9}$.

Values of the dual variables:

$y_1 =$
$y_2 =$
$y_3 =$
$y_4 =$
$y_5 =$
$y_6 =$

Problem 3 (20 points): Change of resources
Assume the daily production of alkylate gasoline increases by 45 barrels to 4,225 barrels. Use your computation of Problem 2 to determine the increase in the total profit for the oil refinery when it establishes a new optimal plan?
Remark: If you feel uncertain about your solution of Problem 2, you should explain very detailed what you are doing in this problem.
Alternative to Problem 3: State the theorem that makes a short answer possible for Problem 3.