Linear algebra and differential equations - 2243
Midterm II
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Teaching Assistant:

Recitation section:

Name:

#	Points
1	/10
2	/10
3	/20
4	/15
5	/15
6	/20
7	/10
Sum	/100
Grade	

Problem 1 (10 points): Complex trial solutions
Use a complex-valued trial solution to determine a particular solution to the given differential equation.

$$y'' + y' - 2y = -80\cos 2x$$

Problem 2 (10 points): Spring-mass system

Determine the motion of the spring-mass system governed by the given initial value problem. State, whether the motion is under-damped, critically damped or over-damped.

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 0, y(0) = 1, \frac{dy}{dt}(0) = 0$$

Problem 3 (20 points): Row echelon form, reduced row echelon form

Use elementary row operations to reduce the given matrices to row echelon form, and then to reduced row echelon form. Determine the rank of the matrices. Label each elementary row operation with the operation symbol for the operation you are performing. E.g., P_{12}, $A_{13}(4)$, or $M_2(-1)$.

a) $\begin{bmatrix} 5 & 4 & -1 & 6 \\ 1 & 2 & 1 & 0 \\ -3 & 1 & 4 & -7 \end{bmatrix}$ b) $\begin{bmatrix} 2+i & -1 \\ 3 & i \end{bmatrix}$

Problem 4 (15 points): Systems of linear equations
Use the inverse of a matrix to determine a solution to the given system of linear equations.

$$\begin{aligned} 3x_1 + 4x_2 + 5x_3 &= 1 \\ 2x_1 + 10x_2 + x_3 &= 1 \\ 4x_1 + x_2 + 8x_3 &= 1 \end{aligned}$$

Problem 5 (15 points): Cramer's rule

Use Cramer's rule to solve the given system of linear equations.

$$\begin{aligned} 3x_1 - 2x_2 + x_3 &= 4 \\ x_1 + x_2 - x_3 &= 2 \\ x_1 + x_3 &= 1 \end{aligned}$$

Problem 6 (20 points): Vector spaces and vector subspaces
Let $V = C^0(I) = \{f \mid f : I \to \mathbb{R} \text{ a continuous function}\}$ the set of all real valued continuous functions defined on an interval $I = [a, b]$ of the reals, and $F = \mathbb{R}$ be a set of scalars. Let addition of elements in V be defined by $(f + g)(x) = f(x) + g(x)$ and scalar multiplication by $(k \cdot f)(x) = k \cdot f(x)$ for $f, g \in V$, $k \in F$, and $x \in I$.

a) Exemplify that V is a vector space over the reals $F = \mathbb{R}$ by showing *closure under addition, closure under scalar multiplication,* and *any three other vector space axioms of your choice* for V.
b) Show that the set $S = \{f \in V : f(a) = f(b)\}$ of all functions in V that take the same values at a and b is a vector subspace of V.

Problem 7 (10 points): Spanning vectors
Let S be the subspace of the vector space $\mathbb{R}^3$ consisting of all solutions to the linear equation $2x_1 - x_2 + x_3 = 0$. Determine a set of vectors that spans S.