Linear algebra and differential equations - 2243
Midterm III
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Teaching Assistant:

Recitation section:

Name:

#	Points
1	/20
2	/15
3	/20
4	/15
5	/15
6	/15
Sum	/100
Grade	

Problem 1 (20 points): Linear independence

a) Determine whether the given set of vectors is linearly independent or dependent in $\mathbb{R}^3$. In case of linear dependence, find a dependency relationship.

$$\{(4,-1,2),(-1,-8,7),(3,2,-1)\}.$$

b) Determine whether the following vectors are linearly independent in P_3. In case of linear dependence, find a dependency relationship.

$$\{x^2-1,x+1,x-1\}.$$

c) Use the Wronskian to determine whether the given functions are linearly independent on the interval $[0,1]$. In case of linear dependence, find a dependency relationship.

$$\{\sin 2x,\cos 2x\}.$$

Problem 2 (15 points): Bases
Determine a basis for the subspace of $M_2(\mathbb{R})$ spanned by

$$\left\{ \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}, \begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix}, \begin{bmatrix} 3 & 11 \\ 14 & 6 \end{bmatrix} \right\}.$$

Problem 3 (20 points): Linear transformations
a) Determine the matrix of the given linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$.

$$T(x_1, x_2, x_3) = (3x_1 - 2x_2 + 5x_3, x_2 - 7x_3).$$

b) Let $T : P_2 \to P_2$ be the linear transformation given by

$$T(1 + x) = 1 - x, T(1 - 3x) = 4.$$

Determine $T(ax + b)$ for arbitrary real numbers a and b.

Problem 4 (15 points): Reflection
Determine the matrix A for the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(\vec{x}) = A\vec{x}$ when T is the reflection along the axis $x = -y$.

Problem 5 (15 points): Eigenvalues, and eigenvectors
Determine all eigenvalues and corresponding eigenvectors of the given matrices. Determine if they are defective or not.

$$\text{a)} \quad \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} \qquad \text{b)} \quad \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

Problem 6 (15 points): Systems of differential equations and diagonalization

Determine a solution of the given initial value problem.

$$\begin{aligned} x_1' &= -6x_1 + 6x_2 \\ x_2' &= -12x_1 + 11x_2 \\ x_1(0) &= 1 \\ x_2(0) &= 1 \end{aligned}$$