Math 1151 Midterm II, October 23, 2000, 9:05 - 9:55

Lecturer: Mark de Longueville

Name:

ID number:

Recitation section #:

Recitation instructor:

Room in which you are right now:

Seat # (if there is one):

Signature:

This exam as the final exam is closed book, no tables, crib sheets, or other aids except for a **low-end scientific calculator.** This is established department policy and practice.
Do one problem after the other. Write all solutions onto this paper, since this is the one going to be graded. You can of course use some scratch paper to make computations. Take your time, do not rush. And do not worry!

1	2	3	4	5	6	7
/24	/15	/10	/6	/10	/20	/15

Sum: /100

Grade:

Exercise 1: Establishing identities Establish each identity (8+8+8 points):

- $\cos^2\theta(1+\tan^2\theta) = 1$

- $\frac{1-\cos\theta}{\sin\theta} + \frac{\sin\theta}{1-\cos\theta} = 2\csc\theta$

- $\frac{1-\sin\theta}{\sec\theta} = \frac{\cos^3\theta}{1+\sin\theta}$

Exercise 2: Sum formulas a) Mark the correct formula (5+5 points):

- $\sin(\alpha+\beta) = \begin{cases} \bigcirc & \sin\alpha\cos\beta - \cos\alpha\sin\beta \\ \bigcirc & \sin\alpha\cos\beta + \cos\alpha\sin\beta \\ \bigcirc & 2\sin\alpha\cos\beta \end{cases}$
- $\cos(\alpha+\beta) = \begin{cases} \bigcirc & \sin\alpha\sin\beta - \cos\alpha\cos\beta \\ \bigcirc & \cos\alpha\cos\beta + \sin\alpha\sin\beta \\ \bigcirc & \cos\alpha\cos\beta - \sin\alpha\sin\beta \end{cases}$

b) Fill in the blank space (5 points):

- $\cos(\alpha-\beta) =$ _____.

Exercise 3: Using a sum formula Find the exact value of

$$\cos 80^\circ \cos 50^\circ + \sin 80^\circ \sin 50^\circ.$$

First write down the sum formula you want to use and then compute the exact value (10 points).

Exercise 4: A double angle formula Use the sum formula

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

to establish a formula for $\tan(2\theta)$ (5 points).

Exercise 5: Using the double angle formula for the tangent function Use the formula for $\tan(2\theta)$ from Exercise 4 or one that you know to establish the following identity (10 points).

$$\frac{\sin(2\theta)}{\sin\theta} = \frac{\cos(2\theta)}{\cos\theta} \cdot \frac{2}{(1-\tan\theta)(1+\tan\theta)}$$

Exercise 6: Solving equations Solve the following equations on the interval $0 \leq \theta < 2\pi$ (10+10 points).

- $\tan(2\theta) = -1$

- $\cos^2\theta - \frac{3\sqrt{2}}{2}\cos\theta + 1 = 0$

Exercise 7: Writing a trigonometric expression as an algebraic expression Show that the equation

$$\sec(\tan^{-1} v) = +\sqrt{1+v^2}$$

holds. You might want to use the identity

$$\csc^2 \alpha = 1 + \cot^2 \alpha.$$

Do not forget to reason why there is +-sign in front of the square root (10+5 points).